ALGEBRAIC IDENTITIES –

An identity is an equality which is true for all values of the variables. Some important identities are

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(v) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(vi) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(vii) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(viii) $a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$

(ix) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Special case:

(i) if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.

(ii) $a^2 + b^2 + c^2 - ab - bc - ac = 0 \rightarrow a = b = c$

Value Form:

(i) $a^2 + b^2 = (a + b)^2 - 2ab$, if $a + b$ and ab are given

(ii) $a^2 + b^2 = (a - b)^2 + 2ab$ if $a - b$ and ab are given

(iii) $a + b = \sqrt{(a-b)^2 + 4ab}$ if $a - b$ and ab are given

(iv) $a - b = \sqrt{(a+b)^2 - 4ab}$ if $a + b$ and ab are given

(v) $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$ if $a + \frac{1}{a}$ is given

(vi) $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 + 2$ if $a - \frac{1}{a}$ is given

(vii) $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ if $(a + b)$ and ab are given

(viii) $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$ if $(a - b)$ and ab are given

(ix) $x^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$ if $a + \frac{1}{a}$ is given

(x) $a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3\left(a - \frac{1}{a}\right)$, if $\left(a - \frac{1}{a}\right)$ is given

(xi) $a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2 = [(a + b)^2 - 2ab]^2 - 2a^2b^2$, if $(a + b)$ and ab are given

(xii) $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = [(a + b)^2 - 2ab](a + b)(a - b)$

(xiii) $a^5 + b^5 = (a^3 + b^3)(a^2 + b^2) - a^2b^2(a + b)$

SINGLE OPTION CORRECT

1. Which of the following are the rational roots of the polynomial $2x^3 + 3x^2 - 11x - 6 = 0$.

   (A) $-2, 3, -\frac{1}{2}$ (B) $2, -3, -\frac{1}{2}$ (C) $2, 3, \frac{1}{2}$ (D) $-2, -3, \frac{1}{2}$

2. If $x + a$ is a factor of $x^4 - a^2x^2 + 3x - 6a$, then the value of 'a' is

   (A) 1 (B) 2 (C) 0 (D) 3

3. Find the remainder when the polynomial $p(x) = x^{100} - x^{97} + x^3$ is divided by $x + 1$ is

   (A) 5 (B) 3 (C) 1 (D) 6

4. Which of the following is a factor of $x^{10} - 1$ and also $x^{11} - 1$?

   (A) $x - 1$ (B) $x + 1$ (C) $x - 10$ (D) $x + 10$

5. If $x^2 - x - 42 = (x + k)(x + 6)$, then the value of k is

   (A) 6 (B) -6 (C) 7 (D) -7

6. The remainder, when the polynomial $x^3 - ax^2 + 6x - a$ is divided by $(x - 1)$ is

   (A) $7 - 2a$ (B) 7 (C) $7 - a$ (D) $7 = 2a$

7. Zero of the zero polynomial is

   (A) 0 (B) 1 (C) Any real number (D) Not defined

8. If the product of $x^2 - 6x + 5$ and $2x^2 - 7x + 3$ is 0, which of the following is not a value of 'x'?

   (A) 3 (B) 2 (C) $\frac{1}{2}$ (D) 1

9. $(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$ is

   (A) $(x - 2y)(2y - z)(3z - x)$ (B) $3(2x - 2y)(2y - 3z)(3z - x)$

   (C) $3(x - 2y)(3y - 2z)(3z - x)$ (D) $3(x - 2y)(2y - 3z)(3z - x)$

10. The expression $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 2x + a$ leave the same remainder when divided by $(x - 2)$, find the value of a.

    (A) 0 (B) 1 (C) 2 (D) -1

11. Factorization of $a^6 - 26a^3 - 27$ is _____

    (A) $(a+3)(a-1)(a^2+3a+1)(a^2-a+9)$ (B) $(a-3)(a+1)(a^2+3a+1)(a^2-a+9)$

    (C) $(a-3)(a+1)(a^2+3a+9)(a^2-a+1)$ (D) $(a-1)(a+3)(a^2+3a+9)(a^2-a+1)$

12. If $x^3+y^3+z^3=3xyz$ and x + y + z = 0. The value of $\frac{(x+y)^2}{xy}+\frac{(y+z)^2}{yz}+\frac{(z+x)^2}{zx}$ is _____

(A) 3 (B) - 3 (C) 2 (D) None of these

13. If $p(x)=x^2-3\sqrt{2}x+1$, then $p\left(5\sqrt{2}\right)$ is equal to _____

(A) $10\sqrt{2}-1$ (B) $10\sqrt{2}+1$ (C) 71 (D) 21

14. The remainder when we divide $p(x) = 4x^4 - 3x^3 + 2x - 1$ by $x^2 + 1$ is

(A) 3x - 5 (B) 3x + 5 (C) 5x + 3 (D) None of these

15. Consider $4x+\frac{6}{y}=15$ & $3x-\frac{4}{y}=7$. If y = ax – 4 then value of a is _____

(A) 7/2 (B) 2 (C) 4/3 (D) - 2

MULTIPLE OPTIONS CORRECT

1. Which of the following is/are not polynomial?

(A) $x^3-4x^2+5\sqrt{x}+1$ (B) $x^{-3}+5x^2+2$ (C) $x^{-2}+4$ (D) x

2. The values of a and b if $x^3-ax^2-13x+b$ has $(x-1)$ and $(x+3)$ as factors

(A) $a=3$ (B) $b=12$ (C) $a=4$ (D) $b=15$

3. Select the correct statement(s)

(A) If p(a) = 0 then (x - a) is a factor of p(x).

(B) Remainder when polynomial p(x) divided by (x – a) is p(a).

(C) Remainder, when $x^3 - 2x + 1$ is divided by 2x – 1 is 1/8.

(D) (x + 2) is a factor of $x^3 + x^2 - x + 2$.

4. Which of the following is/are True?

(A) $(x+a)$ is a factor of a polynomial p(x) iff $p(-a)=0$.

(B) $(ax-b)$ is a factor of a polynomial $p(x)$ iff $p\left(\frac{b}{a}\right)=0$.

(C) $(ax+b)$ is a factor of a polynomial $p(x)$ iff $p\left(\frac{b}{a}\right)=0$.

(D) $(x-a)(x-b)$ is a factor of a polynomial $p(x)$ iff $p(a)=0$ and $p(b)=0$.

5. Select the correct statement(s)

(A) $73^3 - 80^3 + 7^3 = -3 \times 73 \times 80 \times 7$ (B) $x^3 - 1 = (x+1)(x^2 - x + 1)$

(C) $a^2 + b^2 + c^2 - ab - bc - ca = 0 \rightarrow a = b = c$

(D) $(x - a)(x - b)(x - c) = x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc$

6. Which of the following is/are False?

(A) Highest power of the variable in a polynomial is the degree of polynomial.

(B) Degree of zero polynomial is always defined.

(C) A polynomial of degree one is called a constant polynomial.

(D) A polynomial of degree one is called a constant polynomial.

7. Which of the following is/are True. $f(x) = ax^2 + bx + c$.

(A) If f(x) is factorizable into two distinct linear factors then the coordinates are $(-\frac{b}{2a}, -\frac{D}{4a})$

(B) If f(x) is factorizable into two distinct linear factors then the coordinates are $(-\frac{b}{2a}, 0)$

(C) If f(x) is factorizable into two equal factors then the coordinates are $(-\frac{b}{2a}, 0)$

(D) If f(x) is not factorizable then the f(x) opens upwards and remains completely above x-axis, if $a > 0$

8. Which of the following is/are not polynomial

(A) $\frac{x^2}{2} - \frac{2}{x^2}$ (B) $\sqrt{2x} - 1$ (C) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$ (D) $\frac{x-1}{x+1}$

9. Which of the following Statements are not False?

(A) If $f(x)$ is a polynomial with integral coefficients and the leading coefficient is 1, then any integer root of $f(x)$ is a factor of the constant term.

(B) Let $\frac{b}{c}$ be a rational fraction in lowest terms. If $\frac{b}{c}$ is a root of the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \neq 0$ with integral coefficients. Then, b is a factor of constant term a_0 and c is a factor of the leading coefficient a_n.

(C) An nth degree polynomial can have atmost $n + 1$ real roots

(D) Finding a zero or root of a polynomial $f(x)$ means solving the polynomial equation $f(x) = 0$.

10. When $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $g(x)$ gives remainder r then which of the following is/are True?

(A) $g(x) = x + 1, r = 0$ (B) $g(x) = x, r = 1$ (C) $g(x) = x - \frac{1}{2}, r = \frac{27}{8}$ (D) $g(x) = 5 + 2x, r = -\frac{27}{8}$

11. Which of the following statement is/are True?

(A) $a^n + b^n$ is divisible by $a + b$ if $n = 2k + 1$, where k is a positive integer.

(B) $a^n - b^n$ is divisible by $a - b$ if $n = 2k$, where k is a positive integer.

(C) Let $p(x)$ be any polynomial of degree greater than or equal to one and a be any real number. If $p(x)$ is divisible by $(x - a)$, then the remainder is equal to $p(a)$.

(D) When $p(x) = x^4 - 3x^2 + 2x + 1$ is divided by $x - 1$ gives remainder -1.

12. Which of the following statement is/are Correct?

(A) If $x - 2$ is a factors of $x^3 - 2kx^2 + kx - 1$, then $k = \frac{5}{2}$.

(B) If $x - 2$ is a factors of $x^5 - 3x^4 - kx^3 + 3kx^2 + 2kx - 4$, then $k = \frac{7}{6}$.

(C) If $x - 2$ is a factors of $x^3 - kx^2 + 6x - k$, then $k = \frac{7}{3}$.

(D) If $x - 2$ is a factors of $2x^3 + kx^2 + 11x + k + 3$, then $k = -4$.

13. Select the correct factorizations

(A) $4(x+y)^2 - 3(x+y) = (x+y)(4x+4y-3)$

(B) $12 - (x+x^2)(8-x-x^2) = (1-x)(2+x)(2-x)(3-x)$

(C) $16a^4 - b^4 = (4a^2 + b^2)(2a-b)(2a+b)$

(D) $a^2 + \frac{1}{a^2} - 18 = \left(a - \frac{1}{a} + 4\right)\left(a + \frac{1}{a} - 4\right)$

14. Consider $N = 25^3 + 50^3 - 75^3 = -2^x \cdot 3^y \cdot 5^z$. Then

(A) x + y + z = 9

(B) 2x + 2y = z

(C) Rationalizing factor of $y + \sqrt{z}$ is $3 - \sqrt{6}$

(D) Number of zeroes at the end of N when express in decimal system is 1

Olympiad TYPE (Higher Order Thinking)

1. If $x + y + z = 1$, $xy + yz + zx = -1$ and $xyz = -1$, then find the value of $x^3 + y^3 + z^3$.
2. If $\frac{2x}{3} - \frac{4y}{5} = 4$ and $xy = 60$, then find the value of $\frac{4}{9}x^2 + \frac{16}{25}y^2$?
3. Without actual division prove that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.
4. The perimeter of a right triangle is 24 centimeters. Three times the length of the longer leg minus two times the length of the shorter leg exceeds the hypotenuse by 2 centimeters. What are the lengths of all three sides?
5. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, then show that $a = b = c$.
6. If $(3x - 1)^4 = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, then find the value of $a_4 + 3a_3 + 9a_2 + 27a_1 = 81a_0$.
7. If $Ax^3 + 31x^2 - Bx - 10$ is exactly divisible by $2x^2 + 9x - 5$, then Find the values of A and B.
8. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then find the value of q
9. If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $x - 1$ and $x + 1$, the remainders are respectively 5 and 19. Determine the remainder when $f(x)$ is divided by $(x - 2)$.
10. If $(x - 2)$ is a common factor of $x^3 - 4x^2 + ax + b$ and $x^3 - ax^2 + bx + 8$, then find the values of a & b.

SUBJECTIVE PROBLEMS

1. Find the zeros of the following polynomials
   (i) $x^2 + 7x + 10$ (ii) $x^2 - 25$
2. Factorize:
   (i) $27\ a^3b^3 - 45\ a^4b^2$ (ii) $9\ x^2 - 16\ y^2$
   (iii) $x^3 - x$ (iv) $y^2 - 4y + 3$
   (v) $6 - x - x^2$ (vi) $4x^4 + 7x^2 - 2$
   (vii) $(p - q)^3 + (q - r)^3 + (r - p)^3$ (viii) $x^3 - 3x^2 - 9x - 5$
   (ix) $x^2 + 7x^2 - 21x - 27$
3. Factorise:
   (i) $3x^2 + 27y^2 + z^2 - 18xy + 6\sqrt{3}yz - 2\sqrt{3}zx$ (ii) $27\ x^2 + 125y^3$
   (iii) $(2a - 3b + c)^2$ (iv) $\frac{1}{64}a^3 + b^2 + 125\ c^3 - \frac{15}{4}a\ b\ c$
4. Using factor theorem, Show that (a - b) is the factor of $a\ (b^2 - c^2) + b\ (c^2 - a^2) + c\ (a^2 - b^2)$
5. For what value of a is $2x^3 + ax^2 + 11x + a + 3$ exactly divisible by $(2x - 1)$
6. If x - 2 is a factor of a polynomial $f(x) = x^5 - 3x - ax^3 + 3ax^2 + 2ax + 4$, then find the value of a.
7. Find the value of a and b so that $x^2 - 4$ is a factor of $ax + 2x^3 - 3x^2 + bx - 4$
8. Find the value of $x^3 + y^3 + z^3 - 3xyz$, if $x + y + z = 12$ and $x^2 + y^2 + z^2 = 70$

9. Find the value of a & b so that polynomial $x^3 - ax^2 - 13x + b$ is exactly divisible by $(x - 1)$ as well as $(x + 3)$.

10. The polynomial $x^3 - mx^2 + 4x + 6$ when divided by $(x + 2)$ leaves remainder 14 find the value of m

11. The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $(x - 4)$ leave the remainders R_1 & R_2 respectively. Find the values of a in each of the following cases, if

    (i) $R_1 = R_2$ (ii) $R_1 + R_2 = 0$ (iii) $2R_1 - R_2 = 0$

12. $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ leaves remainder 5 and 19 on division by $(x - 1)$ and $(x + 1)$ respectively. Find the remainder when $f(x)$ is divided by $(x - 2)$.

13. If both $(x - 2)$ and $(x - \frac{1}{2})$ are factors of $px^2 + 5x + r$, show that $p = r$.

14. Factorize $x^3 + 13x^2 + 32x + 20$, if it is given that $(x + 2)$ is its factor.

15. Using factor theorem, factorize the polynomial $x^4 + x^3 - 7x^2 - x + 6$.

16. Factorize: $2x^4 + x^3 - 14x^2 - 19x - 6$.

17. Find the remainder when $f(x) = x^{45}$ is divisible by $x^2 - 1$.

18. If $f(x) = x^6 - 10x^5 - 10x^4 - 10x^3 - 10x^2 - 10x + 10$, then find the value of $f(11)$.

19. Find the sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$.

20. Use factor theorem to verify that $x + a$ is a factor of $x^n + a^n$ for any odd positive integer.

21. Find the value of k if the polynomial $f(x) = 2x^3 + kx^2 - 7x - 12$ is divisible by $g(x) = x + 4$, Hence find all the factors of $f(x)$.

22. If $x = 0$ and $x = -1$ are the roots of the polynomial $f(x) = 2x^3 - 3x^2 + ax + b$, find the value of a & b.

23. If $a + b + c = 5$ and $ab^3 + bc + ca = 10$, then find $a^3 + b^3 + c^3 - 3abc$

24. Find the remainder when $p(x)$ is divided by $g(x)$ where $p(x) = x^3 - 6x^2 + 2x - 4$, $g(x) = 1 - \frac{3}{2}x$.

    (By using remainder theorem)

25. If $49x^2 - b = (7x + \frac{1}{2})(7x - \frac{1}{2})$, then find the value of b

26. Let A and B are the remainders when the polynomial $y^3 + 2y^2 - 5ay - 7$ and $y^3 + ay^2 - 12y + 6$ are divided by $y + 1$ and $y - 2$ respectively. If $2A + B = 6$, find the value of a.

27. If k and 2k are zeros of $f(x) = x^3 + 4x^2 + 9kx = 90$, find k and all three zeros of $f(x)$.

28. Find the factors of $x^8 + x^4 + 1$.

29. For what value of k, will the expression $(3x^3 - kx^2 + 4x + 16)$ be divisible by $(x - \frac{k}{2})$?

30. If $a + b + c = 0$, then find the value of $\frac{a^2}{a^2 - bc} + \frac{b^2}{b^2 - ca} + \frac{c^2}{c^2 - ab}$.

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ANSWER KEY & SOLUTION

SINGLE OPTION CORRECT

1. B	2. C	3. C	4. A
5. D	6. A	7. C	8. B
9. D	10. B	11. C	12. A
13. D	14. C	15. B	

MULTI OPTIONS CORRECT

1. A, B, C	2. A, D	3. A, B, C, D	4. A, B, D
5. A, C, D	6. B, C, D	7. A, C, D	8. A, B, D
9. A, B, D	10. A, B, C, D	11. A, B, C	12. A, B, D
13. A, C	14. A, B, D		

OLYMPIAD TYPE

1. 1	2. 80	3.	4. 8, 6, 10
5.	6. 0	7. $A = 6$ & $B = -3$.	8. $\frac{49}{4}$
9. 10	10. 4 & 0		

SUBJECTIVE

11. $1, -\frac{153}{65}, \frac{18}{127}$ 12. 10 13.

14. $(x+2)(x+1)(x+10)$ 15. $(x-1)(x+1)(x-2)(x+3)$

16. $(x+1)(x+2)(x-3)(2x+1)$ 17. x 18. 21

19. - 4 20.

21. $(x+1)$, $(x+4)$ & $(2x-3)$ are the factors of $f(x)$. 22. $a = -5, b = 0$.

23. - 25 24. $-\frac{136}{27}$ 25. $\frac{1}{4}$ 26. 2

27. $k = -3, roots = -3, 6, -5$. 28. $(x^4 + 1 - x^2)(x^2 + 1 + x)(x^2 + 1 - x)$.

29. - 4 30. 2