SECTION – A (MATHEMATICS)

PART - I

SINGLE OPTION CORRECT (+ 4, - 1, 0)

1. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the

   (A) Length of latus rectum 3/2 (B) directrix $4x = -3$

   (C) Length of latus rectum 2 (D) directrix $4x = 3$

2. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is:

   (A) $4\sqrt{6}+\frac{28}{3}$ (B) $4\sqrt{6}-\frac{44}{3}$ (C) $4\sqrt{6}+\frac{44}{3}$ (D) $4\sqrt{6}-\frac{28}{3}$

3. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is :

   (A) $\frac{3\sqrt{5}}{2}$ (B) $3\sqrt{5}$ (C) $\frac{9}{\sqrt{5}}$ (D) $5\sqrt{3}$

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4. If the maximum distance of normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1, b<2$, from the origin is 1, then the eccentricity of the ellipse is :

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{4}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{2}}$

5. Let H be the hyperbola, whose foci are (1 ± √2, 0) and eccentricity is √2. Then the length of its latus rectum is

(A) 3/2 (B) 2 (C) 3 (D) 5/2

6. If $\left(a,\frac{1}{a}\right),\left(b,\frac{1}{b}\right),\left(c,\frac{1}{c}\right)$ & $\left(d,\frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, (abcd) is equal to

(A) 4 (B) 1/4 (C) 1 (D) 16

7. x - 2y + 4 = 0 is a common tangent to $y^2=4x \& \frac{x^2}{4}+\frac{y^2}{b^2}=1$. Then the value of b and the other common tangent are given by:

(A) $b=\sqrt{3}$; x + 2y + 4 = 0 (B) b = 3; x + 2y + 4 = 0

(C) $b=\sqrt{3}$; x + 2y - 4 = 0 (D) $b=\sqrt{3}$; x - 2y - 4 = 0

8. Let a hyperbola whose center is at origin. A line x + y = 2 touches this hyperbola at P (1, 1) and intersect the asymptotes at A and B such that AB = 6√2. Equation of asymptote is

(A) $2x^2 + 5xy + 2y^2 = 0$ (B) $3x^2 + 6xy + 4y^2 = 0$

(C) $2x^2 - 5xy + 2y^2 = 0$ (D) None of these

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9. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the point P and Q. If these tangents intersect at the point T (0, 3) then the area (in sq. units) of ΔPTQ is -

(A) $54\sqrt{3}$ (B) $60\sqrt{3}$ (C) $36\sqrt{5}$ (D) $45\sqrt{5}$

10. Two sets A and B are as under $A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;

$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$. Then: -

(A) $A \subset B$ (B) $A \cap B = \phi$ (an empty set)

(C) neither $A \subset B$ nor $B \subset A$ (D) $B \subset A$

11. Locus of the feet of the perpendiculars drawn from either focus on a variable tangent to the hyperbola $16y^2 - 9x^2 = 1$ is

(A) $x^2 + y^2 = 9$ (B) $x^2 + y^2 = 1/9$ (C) $x^2 + y^2 = 7/144$ (D) $x^2 + y^2 = 1/16$

12. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is:

(A) 7/2 (B) 4 (C) 9/2 (D) 6

13. Let $\lambda x - 2y = \mu$ be a tangent to the hyperbola $a^2x^2 - y^2 = b^2$. Then $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is equal to:

(A) - 2 (B) - 4 (C) 2 (D) 4

14. If the normal at the point P(θ) to the ellipse $5x^2 + 14y^2 = 70$ intersect it again at the point Q(2θ), then

(A) $\cos\theta = -\frac{2}{3}$ (B) $\sin\theta = -\frac{2}{3}$ (C) $\sin\theta = -\frac{1}{3}$ (D) $\cos\theta = -\frac{1}{3}$

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15. Equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at $(0,3)$ is

(A) $x^2+y^2-6y-7=0$ (B) $x^2+y^2-6y+7=0$ (C) $x^2+y^2-6y-5=0$ (D) $x^2+y^2-6y+5=0$

16. If a tangent of slope 'm' at a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through (2a, 0). If e is eccentricity of conic, then

(A) $m^2+e^2=1$ (B) $2m^2+e^2=1$ (C) $3m^2+e^2=1$ (D) $m^2+2e^2=1$

17. The eccentricity of conic $4x^2-9y^2=36$, is

(A) $\frac{\sqrt{13}}{4}$ (B) $\frac{\sqrt{5}}{3}$ (C) $\frac{\sqrt{13}}{2}$ (D) $\frac{\sqrt{13}}{3}$

18. For different values of α, the locus of point of intersection of two straight lines $\sqrt{3}x-y-4\sqrt{3}\alpha=0$ and $\sqrt{3}\alpha x+\alpha y-4\sqrt{3}=0$ is

(A) A hyperbola with e = 2 (B) an ellipse with e = $\sqrt{\frac{2}{3}}$

(C) A hyperbola with e = $\sqrt{\frac{19}{16}}$ (D) an ellipse with e = $\frac{3}{4}$

19. The equation $2x^2+3y^2-8x-18y+35=k$ represents

(A) no locus if k > 0 (B)an ellipse if k < 0 (C) a point if k = 0 (D) a hyperbola if k > 0

20. For the hyperbola $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1,\ \left(0<\alpha<\frac{\pi}{2}\right)$, which one of the following is independent of α

(A) Eccentricity (B)Abscissa of foci (C) directrix (D) vertex

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PART – II

Integer Type (+ 4, -1, 0).

21. The line x = 8 is the directrix of the ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the corresponding focus (2, 0). If the tangent to E at the point P in the first quadrant passes through the point $\left(0,4\sqrt{3}\right)$ and intersects that x-axis at Q then $(3PQ)^2$ equal to

22. Number of points from where two mutually perpendicular tangents can be drawn to hyperbola $\frac{x^2}{5}-\frac{y^2}{7}=1$.

23. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is

24. Let C be the largest circle centered at (2, 0) and inscribed in the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$. If (1, α) lies on C, then $10\alpha^2$ is equal to

25. The vertices of a hyperbola H are (±6, 0) and its eccentricity is $\sqrt{5}/2$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$. If d is the length of the line segment of N between H and the y-axis then d^2 is equal to

26. If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha+5)^2+(16\beta^2+50)^2$ equals _____

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27. Let the hyperbola H: $\frac{x^2}{a^2} - y^2 = 1$ and the ellipse E: $3x^2 + 4y^2 = 12$ be such that the length of latusrectum of H is equal to the length of latus-rectum of E. If e_H & e_E are eccentricities of H and E respectively, then the value of $12\left(e_H^2 + e_E^2\right)$ is equal to _____.

28. For real numbers a, b (a > b > 0), let $\text{Area}\left\{(x,y): x^2 + y^2 \le a^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \ge 1\right\} = 30\pi$ and $\text{Area}\left\{(x,y): x^2 + y^2 \ge b^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\} = 18\pi$. Then the value of $(a - b)^2$ is equal to ____.

29. Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,\ a > 0,\ b > 0$, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4\left(2\sqrt{2} + \sqrt{14}\right)$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then value of $a^2 + b^2$ is equal to _____

30. Let the eccentricity of the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\sqrt{\frac{5}{2}}$ and length of its latusrectum be $6\sqrt{2}$, If y = 2x + c is a tangent to the hyperbola H, then the value of c^2 is equal to _____

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ANSWER KEY

1. D	2. A	3. B	4. C
5. B	6. C	7. A	8. A
9. D	10. A	11. D	12. C
13. D	14. A	15. A	16. C
17. D	18. A	19. C	20. B
21. 39	22. 0	23. 7	24. 118
25. 216	26. 2929	27. 42	28. 12
29. 88	30. 20