

SINGLE OPTION CORRECT

- If $f(x) = \prod_{k=1}^{999} (x^2 - 47x + k)$ then product of all real roots of $f(x) = 0$.
(A) 550! (B) 551! (C) 552! (D) 999!
- A real valued function f satisfying the equation $f(x) + 2f(-x) = x^2 - x$, then sum of real roots of equation $f(f(x)) = 0$ is
(A) -3 (B) -6 (C) -9 (D) -10
- The function $f: \mathbb{R} \rightarrow (1, \infty)$ is defined by $f(x) = 9^{|x|} + (1 + \operatorname{sgn}(x))9^x$ is
(A) one-one and onto (B) Many-one and Onto (C) Many-one and into (D) One-One and onto
- If $f(x)$ is defined for $[-1, 2]$ then the domain of $f([x] - x^2 + 4)$ is where $[.]$ is GIF
(A) $[-\sqrt{3}, \sqrt{3}]$ (B) $[-\sqrt{3}, \sqrt{7}]$ (C) $[-\sqrt{3}, 1] \cup [\sqrt{3}, \sqrt{7}]$ (D) $[-\sqrt{3}, -1] \cup [\sqrt{3}, \sqrt{7}]$
- The function $f(x) = \sqrt{\log_{x^2} x}$ is defined for x belonging to –
(A) $(-\infty, 0)$ (B) $(0, 1)$ (C) $(1, \infty)$ (D) $(0, \infty)$
- Let $f(x)$ be a differentiable function everywhere and satisfies relation $f(x+y) + f(x) + f(y) = 2f(x-y) + 6xy - 4y \forall x, y \in \mathbb{R}$ and $f(0) = -1$, then
(A) $f(x)$ is an even function (B) $f(x)$ is an odd function
(C) $f(1) = 1$ (D) $f(0), f'(4)$ & $f(4)$ are in A.P.
- If $\{x\}$ represents the fractional part of x , then $\left\{\frac{5^{200}}{8}\right\}$ is
(A) $1/4$ (B) $1/8$ (C) $3/8$ (D) $5/8$
- Let $f(x) = \tan x$ and $g(f(x)) = f\left(x - \frac{\pi}{4}\right)$, where $f(x)$ and $g(x)$ are real values functions for all possible value of x , then $f(g(x))$ is
(A) $\tan\left(\frac{x-1}{x+1}\right)$ (B) $\tan(x-1) - \tan(x+1)$ (C) $\frac{f(x)+1}{f(x)-1}$ (D) $\frac{x - \frac{\pi}{4}}{x + \frac{\pi}{4}}$
- If $f(x) = \tan x + \tan^2 x \tan 2x$ and $g(n) = \sum_{r=0}^n f(2^r)$, then $g(2013) - \tan(2^{2014})$ is equal to
(A) 0 (B) $\tan(1)$ (C) $-\tan(1)$ (D) 1
- If $[x]$ and $\{x\}$ denotes the greatest integer and fractional part function, then the number of real x , satisfying the equation $(x-2)[x] = \{x\} - 1$, is
(A) 0 (B) 1 (C) 2 (D) infinite
- The number of real solution of $6^x + 1 = 8^x - 27^{x-1}$ is/are
(A) exactly one (B) Exactly two (C) more than two (D) None of these
- If $[x^3 + x^2 + x + 1] = [x^3 + x^2 + 1] + x$, (where $[.]$ denote the greatest integer function), then number of solutions of the equation $\log_e |[x]| = 2 - |[x]|$ is
(A) 1 (B) 0 (C) 3 (D) 2

SECTION -III

This section contains **1 paragraph of 3 questions**, Each question has **FOUR** options (A), (B), (C) and (D) **ONE OR MORE THAN ONE** of these four option(s) is(are) correct. (*Marking scheme*)

+4 If only the bubble(s) corresponding to all the correct response

0 In none of the bubbles is darkened

-2 In all other cases

If $f : X \rightarrow Y$ be a function defined by $y = f(x)$ such that f is both one-one and onto then there exists a unique function $g : Y \rightarrow X$ such that for each $y \in Y$, $g(y) = x$ iff $y = f(x)$. The function g so defined is called the inverse of f and denoted as $f^{-1}(x) = g(x)$.

- Consider $f : X \rightarrow Y$, $g : Y \rightarrow X$ two invertible functions such that $f^{-1}(x) = g(x)$, then
 - $f(g(x)) = I(x)$, where $I(x)$ = Identity function on X .
 - $(f \circ g)^{-1}(x) = I(y)$, Where $I(y)$ = Identity function on Y .
 - $\frac{g(x_2) - g(x_1)}{x_2 - x_1}$ = May be 4 where x_1, x_2 satisfy the equation $f(x) = g(x)$.
 - Let $h : (1, 2] \rightarrow (0, \sqrt{3}]$, $h(x) = \sqrt{x^2 - 1}$ & $P : \left[-1, 2^{\frac{1}{8}}\right] \rightarrow \left[-1, 2^{\frac{7}{8}}\right]$, $P(x) = x^7$ then domain of $(h \circ P)(x)$ is $\left(1, 2^{\frac{1}{8}}\right]$.
- If $f : \left[\frac{3\pi}{2}, 2\pi\right] \rightarrow [-1, 0]$, where $f(x) = \sin x$ then $f^{-1}(x)$ can Not be
 - $\frac{3\pi}{2} + \sin^{-1} x$
 - $2\pi + \sin^{-1} x$
 - $3\pi - \sin^{-1} x$
 - $\frac{5\pi}{2} - \sin^{-1} x$
- A function $f : I \rightarrow J$ given by $f(i) = j$ where $I = \{0, 1, 2, \dots, 9\}$, $J = \{0, 1, 2, \dots, 100\}$ and i, j are element of set I, J respectively. Then number of bijective functions of type $f : I \rightarrow B$ where $B \subseteq J$ and $f(5) = 5$ is
 - ${}^{100}C_9 \cdot 10!$
 - ${}^{100}C_9 \cdot 9!$
 - ${}^{101}C_{10} \cdot 10!$
 - none of these

MULTI OPTION(S) CORRECT

- Let $\alpha(n) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + (-1)^{n-1} \frac{1}{n}$, then
 - $\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n} = \alpha(2n)$
 - $\alpha(2n) < 1 \forall n$
 - $\alpha(2n) \geq 0.5 \forall n$
 - $0.5 < \alpha(n) < 1 \forall n$
- Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ and $h : \mathbb{R} \rightarrow \mathbb{R}$ be differentiable function such that $f(x) = x^3 + 2x + 1$, $g(f(x)) = x$ and $h(g(g(x))) = x \forall x \in \mathbb{R}$. Then
 - $g'(1) = 1/2$
 - $h'(0) = 10$
 - If $x_0 \in \mathbb{R}$, $x_0^3 + 2x_0 - 2 = 0$ then $h(x_0) = 34$
 - $h(g(2)) = 12$
- $f(x) = \begin{cases} x+a, & x \geq 0 \\ 2-x, & x < 0 \end{cases}$ and $g(x) = \begin{cases} \{x\}, & x < 0 \\ b + \sin x, & x \geq 0 \end{cases}$
 If $f(g(x))$ is continuous at $x = 0$, then which of the following is/are true (where $\{x\}$ represents fractional part function)
 - If $b = 1$, then 'a' can take any real value
 - $b < -1$, is not possible
 - No values of a and b are possible
 - There exist finite ordered pairs (a, b)

4. If $f: \mathbb{R} \rightarrow \mathbb{R}$, such that $f(x+y) + f(x-y) = 2f(x)f(y) \forall x, y \in \mathbb{R}$, then
 (A) $f(x)$ is even function if $f(0) \neq 0$.
 (B) $f(x)$ is odd function if $f(0) \neq 0$.
 (C) $f(x)$ is both even and odd if $f(0) = 0$.
 (D) $f(x)$ is neither even nor odd if $f(0) = 0$.

INTEGER OPTION TYPE (0 - 9)

- Let $A = \{1, 2, 3, 4, 5\}$. If be a bijective function from $A \rightarrow A$ and number of such functions for which $f(k) \neq k; \forall k \in A$, is $11p$, then value of p is _____
- Consider $f(x) = x \sin([x]^4 - 5[x]^2 + 4)$, then number of points in $(-5, 5)$ where $\lim_{x \rightarrow a} f(x) = \text{D.N.E.}$ & $a \in (-5, 5)$?
- Find the number of integral values of x in $[-\pi, \pi]$ which satisfies the domain of $f(x) = \sqrt{\log_2(4\sin^2 x - 2\sqrt{3}\sin x - 2\sin x + \sqrt{3} + 1)}$
- If $f(x) = x \ln(2x) - x$, where $x \in \left[\frac{1}{2e}, \frac{e}{2}\right]$ and range of $f(x)$ is $\left[-\frac{1}{a}, b\right]$ then value of $a + b$ is _____
- The number of integral values of x satisfying $\sqrt{-x^2 + 10x - 16} < x - 2$ is _____
- $f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)}$, where x is a real number $x \neq 0, x \neq 1$, then the value of $f(2)$ must be _____
- If the number of the positive integral solutions of the equation $\left[\frac{x}{9}\right] = \left[\frac{x}{11}\right]$ is n , then $\frac{n}{3}$ is _____
 (where $[.]$ is GIF)
- f is a function such that $f(x) + 2f(1-x) = x^2 + 1$. The value of $f(3)$ is _____
- If $x \in (-1, 0)$ and $f(x) = \frac{1+x+\sqrt{1+x^2}}{1-x+\sqrt{1+x^2}}$, then number of solutions of $\cot(f(x)) = 4$ is _____
- If $x^5 - x^3 + x = a$, when $x > 0$, then the maximum value of $2a - x^6$ is equal to _____
- Number of solutions of the equation $f(x) - f^{-1}(x) = 0$ is/are, when $f(x) = \begin{cases} 1 - \frac{x}{3}, & x \leq 0 \\ 1 - x^2, & x > 0 \end{cases}$ _____
- If $f(x) = 2x^3 - 3x^2 + 1$ then number of distinct real solution(s) of the equation $f(f(x)) = 0$ is/are _____

MATRIX MATCH TYPE

1. Match the following Column - I with Column - II

	COULUMN - I		COULUMN - II
A	Range of $f(x) = \sqrt{x-1} + 2\sqrt{3-x}$ is	P	$\left[\frac{3}{2}, \infty\right) - \{3\}$
B	The superset(s) of domain of function ($[.]$ = G.I.F. & $ $ = Modulus) $f(x) = \log_{\left[x+\frac{1}{2}\right]} \left(x^2 - x - 6 \right) + \left[\sqrt{x-1}\right] - 3\sqrt{\sqrt{19} - \sqrt{2}x}$ is/are	Q	$\left[\frac{3}{2}, \frac{5}{2}\right)$
C	A function $f(x)$ is defined for $\forall x \in (-1, 8]$, then domain of function $g(x) = f(x^2) + f(\sqrt{x-2})$ is a subset of	R	$[\sqrt{2}, \sqrt{10}]$
D	Exactly one root of the equation $2(x-1)^3 - 9(x-1)^2 + 12x - 16 = 0$ lies in the interval	S	$[\sqrt{2}, 2\sqrt{2}]$

2. Match the following

	COULUMN - I		COULUMN - II
A	$f: \mathbb{R} \rightarrow \left[\frac{\pi}{4}, \pi\right)$ & $f(x) = \cot^{-1}(2x - x^2 - 2)$, then $f(x)$ is	P	one-one
B	$f: \mathbb{R} \rightarrow \mathbb{R}$ & $f(x) = e^{ax} \sin(bx)$ where $a, b \in \mathbb{R}^+$, then $f(x)$ is	Q	into
C	$f: \mathbb{R}^+ \rightarrow [2, \infty)$ & $f(x) = 2 + 3x^2$, then $f(x)$ is	R	many-one
D	$f: X \rightarrow X$ & $f(f(x)) = x \forall x \in X$ then $f(x)$ is	S	onto

3. A function is defined as $f: \{a_1, a_2, a_3, a_4, a_5, a_6\} \rightarrow \{b_1, b_2, b_3\}$

	COLUMN - I		COLUMN - II
(A)	The number of onto functions	(P)	Is divisible by 9
(B)	Number of function in which $f(a_i) \neq b_i$	(Q)	Is divisible by 5
(C)	Number of invertible functions	(R)	Is divisible by 4
(D)	Number of many one functions	(S)	Is divisible by 3
		(T)	Not possible

THANKS



ANSWER KEY – RELATION & FUNCTION

SINGLE OPTION CORRECT

- | | | | |
|------|-------|-------|-------|
| 1. C | 2. A | 3. B | 4. D |
| 5. B | 6. D | 7. B | 8. A |
| 9. C | 10. A | 11. B | 12. B |

PARAGRAPH TYPE

- | | | |
|------------|------------|------|
| 1. B, C, D | 2. A, C, D | 3. B |
|------------|------------|------|

MULTI OPTIONS CORRECT

- | | | | |
|-------------|------------|---------|---------|
| 1. A, B & C | 2. A, B, C | 3. A, B | 4. A, C |
|-------------|------------|---------|---------|

INTEGER TYPE

- | | | | |
|------|------|------|------|
| 1. 4 | 2. 6 | 3. 6 | 4. 2 |
| 5. 3 | 6. 3 | | |

MATRIX MATCH

- $A \rightarrow R, B \rightarrow P, R, C \rightarrow R, S, D \rightarrow P, Q, S.$
- $A \rightarrow Q, R, B \rightarrow R, S, C \rightarrow P, Q, D \rightarrow P, S.$
- $A \rightarrow P, Q, R, S, B \rightarrow P, R, S, C \rightarrow T, D \rightarrow P, S$

HINTS & SOLUTION

SINGLE OPTION CORRECT

1. C

Solution:

Consider $x^2 - 47x + k = 0$, For real roots $47^2 - 4k \geq 0 \rightarrow k \leq 552$

$\therefore k = 1, 2, 3, \dots, 552$

Product of real roots $= 1 \times 2 \times 3 \times 4 \times \dots \times 552 = 552!$

2. A

Sol: Replace x by $-x$

$$f(-x) + 2f(x) = x^2 + x$$

$$\text{Solving } f(x) = \frac{x^2 + 3x}{3}$$

$$f(f(x)) = \frac{(f(x))^2 + 3f(x)}{3} = 0$$

$$f(x)(f(x) + 3) = 0 \rightarrow f(x) = 0 \text{ or } f(x) = -3$$

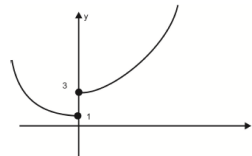
$$\Rightarrow x = 0, -3. \quad x^2 + 3x + 9 = 0 \text{ no solution :}$$

$$\therefore \text{Sum} = -3$$

3. B

Sol:

$$\begin{aligned} 9^x + (1+1)9^x &= 3 \cdot 9^x & x > 0 \\ f(x)9^{-x} + (1-1)9^x &= 9^x & x < 0 \\ 2 & & x = 0 \end{aligned}$$



4. D

$$1 \leq [x] - x^2 + 4 \leq 2 \rightarrow x^2 - 5 \leq [x] \leq x^2 - 2$$

5. B

$$\text{Sol: } \log_{x^2} x \geq 0$$

Now

Case I

$$0 < x^2 < 1$$

$$\Rightarrow -1 < x < 1 \text{ and } x^2 \neq 1 \Rightarrow x \neq -1, 1$$

$$\text{and } 0 < x < 1$$

$$\text{So } x \in (0, 1)$$

Case II

$$x^2 > 1$$

$$\Rightarrow x < -1, \text{ or } x > 1$$

$$\text{and } x^2 \neq 1 \Rightarrow x \neq -1, 1$$

$$\text{and } x > 1$$

$$\Rightarrow x \in (1, \infty)$$

6. D

7. B

$$\frac{5^{200}}{8} = \frac{(1+24)^{100}}{8}$$

$$\therefore \left\{ \frac{5^{200}}{8} \right\} = \frac{1}{8}$$

8. A

$$g(f(x)) = \tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - 1}{\tan x + 1}$$

$$\Rightarrow g(x) = \frac{x-1}{x+1}$$

$$f(g(x)) = \tan\left(\frac{x-1}{x+1}\right)$$

9. C

$$\because f(x) = \tan 2x - \tan x$$

$$\Rightarrow f(2^n) = \tan 2^{n+1} - \tan 2^n$$

$$\Rightarrow g(n) = \tan 2^{n+1} - \tan 1$$

$$g(2013) - \tan 2^{2014} = -\tan 1$$

10. A

for $x \geq 2$, LHS is always non negative and RHS is always -ve

Hence, for $x \geq 2$ no solution

If $1 \leq x < 2$ $(x-2) = (x-1) - 1 = x-2$ which is an identity

For $0 \leq x < 1$, LHS is '0' and RHS is (-)ve $\rightarrow$ no solution

$x < 0$, LHS is (+)ve, RHS is (-)ve $\rightarrow$ no solution

11. B

$$\text{Let } a = 2^x, b = -3^{x-1}, c = -1$$

$$\rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\text{Now, } a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a+b+c)\sum(a-b)^2 \text{ factorizing and get } a+b+c=0 \text{ as } a \neq b \neq c$$

Hence, the given equation is; $2^x - 3^{x-1} - 1 = 0$, By trial $x = 1$ and 2 are solution of this

Now, $2^x = 3^{x-1} + 1$ has no other solution $\rightarrow$ Number of real solution exactly two.

12. B

If $[x^3 + x^2 + x + 1] = [x^3 + x^2 + 1] + x$, then x is integer

$\rightarrow \log_e |[x]| = 2 - |[x]|$ has same solutions as $\log_e |x| = 2 - |x|$, x is integer

$\rightarrow$ No. of integral solution = 0

SECTION -III (PARAGRAPH)

1. B, C, D

2. A, C, D

3. B

MULTI OPTION(S) CORRECT

1. A, B & C

Solution:

$$\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n} = \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2n}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}\right)$$

$$\alpha(2n) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots - \frac{1}{2n} = \left(1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2n-1}\right) - \frac{1}{2} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}\right)$$

$$\alpha(2n) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots - \frac{1}{2n} = \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2n}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}\right)$$

$$\alpha(2n) = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n}$$

$$\text{Also, } \alpha(2n) < \underbrace{\frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n}}_{n \text{ times}} = 1 \text{ and } \alpha(2n) \geq \underbrace{\frac{1}{2n} + \frac{1}{2n} + \dots + \frac{1}{2n}}_{n \text{ times}} = \frac{1}{2}$$

(D) is wrong, as $\alpha(1) = 1$

2. A, B, C

Solution:

$f(x) = x^3 + 2x + 1 \rightarrow f'(x) = 3x^2 + 2 > 0 \forall x \in \mathbb{R}$, so $f(x)$ is strictly increasing and Bijective function.

So $f^{-1}(x)$ will exist.

- (A) $g'(f(x)) f'(x) = 1 \dots\dots\dots (1)$
 $f(0) = 1, f'(0) = 2$ so from (1) we can say, $g'(f(0)) f'(0) = 1 \rightarrow g'(1) \times 2 = 1 \rightarrow g'(1) = 1/2$.
- (B) $h(g(g(x))) = x \rightarrow h'(g(g(x))) g'(g(x)) g'(x) = 1 \dots\dots\dots (2)$
 $g(f(x)) = x \rightarrow g(f(f^{-1}(x))) = f^{-1}(x) \rightarrow g(x) = f^{-1}(x)$.
 $f(0) = 1, f(1) = 4 \rightarrow f^{-1}(4) = 1 \text{ \& } f^{-1}(1) = 0$
 $h'(g(g(4))) g'(g(4)) g'(4) = 1 \rightarrow h'(0) g'(1) g'(4) = 1 \dots\dots\dots (3)$
from (1) we get, $g'(f(0)) f'(0) = 1 \rightarrow g'(1) \times 2 = 1 \rightarrow g'(1) = 1/2$
 $g'(f(1)) f'(1) = 1 \rightarrow g'(4) \times 5 = 1 \rightarrow g'(4) = 1/5$
Now, from (3) we get $h'(0) = 10$
- (C) $x_0^3 + 2x_0 - 2 = 0 \rightarrow f(x_0) = x_0^3 + 2x_0 + 1 = 3 \rightarrow g(3) = x_0$
 $h(x_0) = h(g(3)) = h(g(g(34))) = 34$
- (D) $f^{-1}(x_0) = 2 \rightarrow f(2) = x_0 = 13$ so $h(g(g(13))) = 13 \rightarrow h(g(2)) = 13$.

3. A, B

Solution:

For $b = 1$, we have $f(g(0)) = f(\sin(0) + 1) = f(1) = a + 1$

Also, $f(g(0^+)) = \lim_{x \rightarrow 0^+} f(1 + \sin x) = f(1) = a + 1$

And $f(g(0^-)) = \lim_{x \rightarrow 0^-} f(\{x\}) = f(1^-) = a + 1$

Hence, $f(g(x))$ is continuous for $b = 1$

For, $b < 0$, $f(g(x))$ is continuous at $x = 0$ if $1 = 2 - b$ for which $b = 1$, which is not possible

4. A, C

INTEGER OPTION TYPE (0 - 9)

1. 4

Solution:

Number of required bijective functions = $5! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right) = 44 = 11p(\text{gives}) \rightarrow p = 4$

2. 6

3. 6

$$\log_2 (4 \sin^2 x - 2(\sqrt{3} \sin x + \sin x) + (\sqrt{3} + 1)) \geq 0$$

$$\Rightarrow 4 \sin^2 x - 2 \sin x (\sqrt{3} + 1) + \sqrt{3} + 1 \geq 1$$

$$\Rightarrow \sin^2 x - \sin x \left(\frac{\sqrt{3} + 1}{2} \right) + \frac{\sqrt{3} + 1}{4} \geq 0$$

$$\Rightarrow \left(\sin x - \frac{\sqrt{3} + 1}{2} \right) \left(\sin x - \frac{1}{2} \right) \geq 0$$

$$\Rightarrow -1 \leq \sin x \leq \frac{1}{2} \text{ or } \frac{\sqrt{3} + 1}{2} \leq \sin x \leq 1$$

$$\Rightarrow x \in \left[-\pi, \frac{\pi}{6} \right] \cup \left[\frac{\pi}{3}, \frac{2\pi}{3} \right] \cup \left[\frac{5\pi}{6}, \pi \right]$$

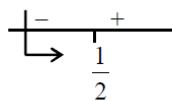
Number of integral solutions $4 + 1 + 1 = 6$

4. 2

$$\Rightarrow f'(x) = \ln 2x + 1 - 1 = \ln 2x$$

$$\Rightarrow f\left(\frac{1}{2}\right) = -\frac{1}{2}, \quad f\left(\frac{e}{2}\right) = \frac{e}{2} \ln e - \frac{e}{2} = 0$$

$$\Rightarrow \text{Range} \left[-\frac{1}{2}, 0 \right] \Rightarrow a + b = 2$$



5. 3

$$\begin{aligned}
 -x^2 + 10x - 16 &\geq 0 \Rightarrow 2 \leq x \leq 8 \\
 -x^2 + 10x - 16 &< x^2 - 4x + 4 \\
 \Rightarrow 2x^2 - 14x + 20 &> 0 \Rightarrow x > 5 \text{ or } x < 2 \\
 \Rightarrow x &= 6, 7, 8
 \end{aligned}$$

6. 3

Replacing x by $\frac{1}{(1-x)}$, we obtain $f\left(\frac{1}{1-x}\right) + f\left(1 - \frac{1}{x}\right) = -2x + \frac{2}{x}$

Again, replacing x by $1 - \frac{1}{x}$

7. 8

$$\left\lfloor \frac{x}{9} \right\rfloor = \left\lfloor \frac{x}{11} \right\rfloor = l$$

$\Rightarrow l \leq \frac{x}{9} < l+1, l \leq \frac{x}{11} < l+1$, Solution is possible only when $11l < 9l+9$

$$\Rightarrow 0 \leq l < \frac{9}{2}$$

Total number of solution $\sum_{l=0}^4 (9-2l) - 1 = 24$

8. 0

9. 0

$$\text{Let } \tan^{-1} x = \theta \Rightarrow \theta \in \left(-\frac{\pi}{4}, 0\right)$$

$$f(x) = \frac{1 + \tan \theta + |\sec \theta|}{1 - \tan \theta + |\sec \theta|} = \frac{\sec \theta + \tan \theta + 1}{\sec \theta - 1 + \tan \theta + 1}$$

$$= \sec \theta + \tan \theta$$

$$= \frac{1 + \sec \theta}{\cos \theta} = \frac{1 + \cos\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right)}$$

$$= \frac{2 \cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{2 \sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right) \cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)} = \cot\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

$$\text{if } \theta \in \left(-\frac{\pi}{4}, 0\right) \Rightarrow \frac{\pi}{4} < \frac{\pi}{4} - \frac{\theta}{2} < \frac{3\pi}{8}$$

$$\therefore \cot^{-1} f(x) = 4$$

$$\Rightarrow \cot^{-1} \cot\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = 4$$

$$\frac{\pi}{4} - \frac{\tan^{-1} x}{2} = 4$$

$$\Rightarrow \frac{\pi - 16}{2} = \tan^{-1} x \notin (-1, 0)$$

10. 1

$$\text{We have } a = \frac{x(x^6 + 1)}{x^2 + 1}$$

$$\Rightarrow x^6 + 1 = a \left(\frac{x^2 + 1}{x} \right) \geq 2a$$

$$\Rightarrow 2a - x^6 \leq 1$$

11. 5

12. 3

MATRIX MATCH TYPE

1. $A \rightarrow R, B \rightarrow P, R \rightarrow C, S \rightarrow D, Q \rightarrow P, Q, S.$
2. $A \rightarrow Q, R \rightarrow B, R, S \rightarrow C, P, Q \rightarrow D, P, S.$
3. $(A) \rightarrow p, q, r, s \quad (B) \rightarrow p, r, s \quad (C) \rightarrow t \quad (D) \rightarrow p, s$