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MATHEMATICS **QUADRATIC EQUATION**

SINGLE OPTION CORRECT

1. If the expression $x^2-(5m-2)x+(4m^2+10m+25)$ can be expressed as a perfect square, then m =

(A) 8/3 or 4 (B) - 8/3 or 4 (C) 4/3 or 8 (D) - 4/3 or 8

2. The value of λ for which one root of the equation $x^2+(1-2\lambda)x+(\lambda^2-\lambda-2)=0$ is greater than 3 and the other is less than 3 is given by

(A) $\lambda < 2$ (B) $2 < \lambda < 5$ (C) $\lambda > 5$ (D) $\lambda > 1$

3. The value of m for the roots of $2x^2 - mx - 8 = 0$ differ by (m – 1) is

(A) 4, - 10/3 (B) - 6, 10/3 (C) 6, 10/3 (D) 6, - 10/3

4. If α and β $(\alpha<\beta)$ are the roots of the equation $x^2+bx+c=0$, where $c<0<b$ then

(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta$ (C) $\alpha < \beta < 0$ (D) Cant Say

5. If the equation $k(6x^2+3)+rx+(2x^2-1)=0$ and $6k(2x^2+1)+px+(4x^2-2)=0$ have both roots common, then the value p/r is

(A) 1/2 (B) 2 (C) 1 (D) 4

6. If $y=2+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+......\infty}}}$

(A) y = 6 (B) y = 5 (C) $y = \sqrt{6}$ (D) $y = \sqrt{5}$

7. The value of $\sqrt{8+2\sqrt{8+2\sqrt{8+2\sqrt{8+.....\infty}}}}$ is

(A) 10 (B) 6 (C) 8 (D) 4

8. If α and β are roots of $4x^2 - 16x + \lambda = 0$ such that $\alpha \in (1, 2)$, $\beta \in (2, 3)$, the sum of all the integral values of λ is

(A) 42 (B) 32 (C) 22 (D) 12

9. If $f(x) = (x - a_1)^2 + (x - a_2)^2 + (x - a_3)^2 + \dots\dots\dots + (x - a_n)^2$. Find x where f(x) is minimum

(A) $-\infty$ (B) $\dfrac{a_1+a_2+a_3+.......+a_n}{n}$

(C) $-\dfrac{a_1+a_2+a_3+.......+a_n}{n}$ (D) none of these

10. If the larger root of equation $x^2 + (2 - a^2)x + (1 - a^2) = 0$ is less than both the roots of the equation $x^2 - (a^2 + 4a + 1)x + a^2 + 4a = 0$, then the range of a, is

(A) $(-\sqrt{2},\sqrt{2})$ (B) $\left(-\frac{1}{4},\sqrt{2}\right)$ (C) $\left(-\sqrt{2},\frac{1}{4}\right)$ (D) none of these

11. If one solution of the equation $x^3 - 2x^2 + ax + 10 = 0$ is the additive inverse of another, then which one of the following inequalities is true?

(A) $-40 < a < -30$ (B) $-30 < a < -20$ (C) $-20 < a < -10$ (D) $-10 < a < 0$

12. The value of $f(x) = x^2 + (p - q)x + p^2 + pq + q^2$ for real values of p, q and x

(A) is always negative (B) is always positive
(C) is some time zero for non zero value of x (D) None of these

13. Solution set for the inequation $\frac{x^2-1}{x} \le 2-x$ is

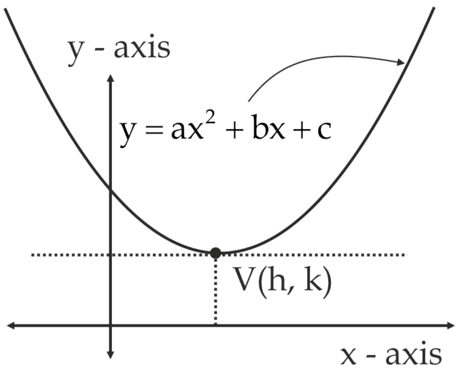
(A) $x \in \left(-\infty, \frac{1-\sqrt{3}}{2}\right] \cup \left(0, \frac{1+\sqrt{3}}{2}\right]$ (B) $x \in \left[\frac{1-\sqrt{3}}{2}, 0\right) \cup \left[\frac{1+\sqrt{3}}{2}, \infty\right)$

(C) $x \in \left[\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right]$ (D) None of these

14. The number of distinct real roots of equation $(|x|-1)^{|x-1|-3} = 1$

(A) 3 (B) 4 (C) 5 (D) None of these

15. Consider the figure of real quadratic $y = Q(x) = ax^2 + bx + c$ as shown. Select the **wrong** option (Where $D = b^2 - 4ac$, $i = \sqrt{-1}$)



(A) One root of the equation $ax^2 + bx + c = 0$ is $x = \frac{-b + i\sqrt{-D}}{2a}$.

(B) $ax^2 + bx + c > 0 \ \forall \ x \in R, a \neq 0$

(C) $|a| + |b| + c = 0$ for at least one real triplet (a, b, c).

(D) $h = -\frac{b}{2a}$ & $k = -\frac{D}{4a}$

16. Solution set of $\frac{|x-1|}{x(x-2)|x-3|} \ge 0$ is

(A) $x \in (-\infty, 0) \cup (2, \infty)$ (B) $x \in (-\infty, 0) \cup (2, \infty) \cup \{1\} - \{3\}$
(C) $x \in (0, 2)$ (D) none of these

17. a cubic polynomial P(x) when divided by (x - 1), (x - 2) and (x - 3) leaves remainder 3, 8 and 15 respectively. If P(4) = 30 then the remainder, when P(x) is divided by (x + 1) is

(A) - 25 (B) - 20 (C) - 16 (D) none of these

18. If $(1+x)(1+x^2)(1+x^4)(1+x^8)......(1+x^{128}) = \sum_{r=0}^{n} x^r$ then n is equal to

(A) 255 (B) 127 (C) 63 (D) None of these

13. The equation $2^{2x} + (a-1)\cdot 2^{x+1} + a = 0$ has roots of opposite sign then exhaustive set of values of 'a' is

(A) $a < 0$ (B) $a \in (-1, 0)$ (C) $a \in (-\infty, 1/3)$ (D) $x \in (0, 1/3)$

14. Let α and β are the roots of the equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is

(A) $\frac{\sqrt{34}}{9}$ (B) $\frac{2\sqrt{13}}{9}$ (C) $\frac{\sqrt{61}}{9}$ (D) $\frac{2\sqrt{17}}{9}$

15. Solution of the equation: $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$ is

(A) $x \in [4, 9]$ (B) $x \in [3, 8]$ (C) $x \in [5, 10]$ (D) $x \in [4, 7]$

16. If the range of $f(x)=\frac{2x^4-14x^2-8x+49}{x^4-7x^2-4x+23}$ is (a, b], then (a + b) is

(A) 3 (B) 4 (C) 5 (D) 6

17. Consider the equation $x^2 + \alpha x + \beta = 0$ having roots α, β such that $\alpha \neq \beta$. Also consider the inequality $\left||y-\beta|-\alpha\right|<\alpha$, then

(A) in-equality is satisfied by exactly two integral values of y

(B) in-equality is satisfied by all values of $y \in (-4, 2)$

(C) Roots of the equation are of same sign

(D) $x^2+\alpha x+\beta>0 \; \forall x \in [-1,0]$

18. If $Q(a) = a^2 + a + 1$, then number of solutions of equation $Q(a^2) = 3\,Q(a)$ is

(A) 0 (B) 1 (C) 2 (D) more than 2

19. If the equation in x, $x^4 + px^3 + qx^2 = 16(2x - 1)$, where $p, q \in R$ has all positive roots, then

(A) $q : |p| = 3 : 2$ (B) $p > 8$ (C) $q \geq 4$ (D) $p < 0 < q < 8$

20. Let α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$. If a, b, c are in A.P. and $\alpha + \beta = 15$, then αβ equals

(A) - 21 (B) - 29 (C) - 31 (D) - 39

21. Let α and β are the roots of $x^2 - \sqrt{2}\,x + 1 = 0$, then the value of $\alpha^{50}+\beta^{50}$ is -

(A) 0 (B) $\sqrt{2}$ (C) 2 (D) 1

22. If the equation $\frac{1}{x}+\frac{1}{x-1}+\frac{1}{x-2}=3x^3$ has k real roots, then k is equal to -

(A) 2 (B) 3 (C) 4 (D) 6

23. Let $f(x) = x^3 + x^2 + 1$; $g(x) = x^2 - 1$. If the roots of f(x) are x_1, x_2 and x_3 then the value of $g(x_1)\cdot g(x_2)\cdot g(x_3)+17g(x_1x_2x_3)$ is -

(A) 3 (B) 7 (C) 17 (D) 20

24. Let r(x) be the remainder when the polynomial $x^{135} + x^{125} - x^{115} + x^5 + 1$ is divided by $x^3 - x$. Then

(A) r(x) is the zero polynomial (B) r(x) is a nonzero constant

(C) degree of r(x) is one (D) degree of r(x) is two

(KVPY - 17)

25. Let A, G and H be the arithmetic mean, geometric mean and harmonic mean, respectively of two distinct positive real numbers. If α is the smallest of the two roots of the equation

$A(G-H)x^2+G(H-A)x+H(A-G)=0$, then

(A) $-2 < \alpha < -1$ (B) $0 < \alpha < 1$ (C) $-1 < \alpha < 0$ (D) $1 < \alpha < 2$

26. The sum of all non-integer roots of the equation $x^5 - 6x^4 + 11x^3 - 5x^2 - 3x + 2 = 0$ is

(A) 6 (B) - 11 (C) - 5 (D) 3

27. Let f(x) be a quadratic polynomial with f(2) = 10 and f(–2) = – 2. Then the coefficient of x in f(x) is :
(A) 1 (B) 2 (C) 3 (D) 4

28. Suppose a, b, c are three distinct real numbers. Let $P(x) = \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} + \frac{(x-a)(x-b)}{(c-a)(c-b)}$
when simplified, P(x) becomes
(A) 1 (B) x
(C) $\frac{x^2+(a+b+c)(ab+bc+ca)}{(a-b)(b-c)(c-a)}$ (D) 0

29. Let a, b, c, d be real numbers such that |a – b| = 2, |b – c| = 3, |c – d| = 4. Then the sum of all possible values of |a - d| is
(A) 9 (B) 18 (C) 24 (D) 30

30. If $x+\frac{1}{x}=a, x^2+\frac{1}{x^3}=b$, then $x^3+\frac{1}{x^2}$ is
(A) $a^3 + a^2 - 3a - 2 - b$ (B) $a^3 - a^2 - 3a + 4 - b$ (C) $a^3 - a^2 + 3a - 6 - b$ (D) $a^3 + a^2 + 3a - 16 - b$

31. In the real number system, the equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$ has –
(A) No solution (B) Exactly two distinct solutions
(C) Exactly four distinct solutions (D) Infinitely may solutions

32. Let a, b, c, d be numbers in the set {1, 2, 3, 4, 5, 6} such that the curves $y = 2x^3 + ax + b$ and $y = 2x^3 + cx + d$ have no point in common. The maximum possible value of $(a - c)^2 + b - d$ is –
(A) 0 (B) 5 (C) 30 (D) 36

33. Let f : R → R be the function $f(x) = (x - a_1)(x- a_2) + (x- a_2)(x- a_3) + (x- a_3)(x- a_1)$ with $a_1, a_2, a_3 \in R$. Then f(x) > 0 if and only if –
(A) At least two of a_1, a_2, a_3 are equal (B) $a_1 = a_2 = a_3$
(C) a_1, a_2, a_3 are all distinct (D) a_1, a_2, a_3 are all positive and distinct

34. A student notices that the roots of the equation $x^2 + bx + a = 0$ are each 1 less than the roots of the equation $x^2+ax+b=0$. Then a + b is:
(A) - 4 (B) - 2 (C) - 4 (D) - 5

35. Let r be a root of the equation $x^2 + 2x + 6 = 0$. The value of (r + 2) (r + 3) (r + 4) (r + 5) is equal to -
(A) 51 (B) - 51 (C) - 126 (D) 126

36. Let $p(x) = x^2 - 5x + a$ and $q(x) = x^2 - 3x + b$, where a and b are positive integers. Suppose hcf (p(x), q(x)) = x – 1 and k(x) = lcm (p(x), q(x)). If the coefficient of the highest degree term of k(x) is 1, the sum of the roots of (x – 1) + k(x) is -
(A) 4 (B) 5 (C) 6 (D) 7

37. Two distinct polynomials f(x) and g(x) are defined as follows: $f(x) = x^2 + ax + 2$; $g(x) = x^2 + 2x + a$. If the equations f(x) = 0 and g(x) = 0 have a common root then the sum of roots of the equation
f(x) + g(x) = 0 is -
(A) - 1/2 (B) 0 (C) 1/ 2 (D) 1

38*. Suppose the quadratic polynomial $P(x) = ax^2 + bx + c$ has positive coefficients a, b, c in arithmetic progression in that order. If P(x) = 0 has integer roots α and β then α + β + α β equals
(A) 3 (B) 5 (C) 7 (D) 14

39. The number of ordered pairs (x, y) of real numbers that satisfy the simultaneous equations $x + y^2 = x^2 + y = 12$ is

(A) 0 (B) 1 (C) 2 (D) 4

40. Consider the quadratic equation $\alpha (x - 1)^2 + x - 3 = 0$. If α is of the form $\frac{k(k+1)}{2}, k \in Q$, then roots of equation are necessarily-

(A) integers (B) imaginary (C) rational numbers (D) can not be predicted

41. Set of all real values of 'a' such that $f(x) = \frac{(2a-1)x^2 + 2(a+1)x + (2a-1)}{x^2 - 2x + 40}$ is always negative is

(A) $(-\infty, 0)$ (B) $(0, \infty)$ (C) $(-\infty, 1/2)$ (D) None of these

42. Set of all values of x satisfying the inequality $\sqrt{x^2 - 7x + 6} > x + 2$ is –

(A) $x \in \left(-\infty, \frac{2}{11}\right)$ (B) $x \in \left(\frac{2}{11}, \infty\right)$ (C) $x \in (-\infty, 1] \cup [6, \infty)$ (D) $x \in [6, \infty)$

43. Suppose that the roots of $x^3 + 3x^2 + 4x - 11 = 0$ are a, b and c, and the roots of $x^3 + rx^2 + sx + t = 0$ are a + b, b + c and c + a. Find t __

(A) 23 (B) 24 (C) 25 (D) 26

MULTIPLE OPTIONS CORRECT

1. The integer value of k for which $(k-2)x^2 + 8x + k + 4 > 0 \, \forall x \in R$ is

(A) 5 (B) 6 (C) 7 (D) 4

2. Find the value of k for which the graph of the quadratic polynomial $P(x) = x^2 + (2x + 3)k + 4(x + 2) + 3k - 5$ intersects the axis of x at two distinct points.

(A) 1 (B) 2 (C) 5 (D) 4

3. Select the correct statement(s) for solution set of x

(A) $|2x - 1| > -1 \rightarrow x \in R$

(B) $\frac{1}{x-1} < x \rightarrow x(x-1) > 1 \; \forall \; x > 1$

(C) $\frac{|x| - 1}{x(x-2)} < 0 \equiv \frac{(x+1)(x-1)}{x(x-2)} < 0$

(D) $|x - 1|(x-2)^2 \le 0 \rightarrow x \in \phi$

4. Select the correct statement(s) for real numbers a, b, c and d.

(A) If ab = 0 and a = 0 then $b \in R$

(B) if ab = ac then $\not{a} b = \not{a} c \rightarrow b = c \; \forall \; a \in R$

(C) $\frac{a^2 b}{c} \ge 0 \rightarrow \frac{b}{c} \ge 0 \;\&\; a \in R$

(D) $\frac{a}{b} \ge \frac{c}{d} \rightarrow ad \ge bc \;\; \forall \;\; b, d \in R^+$

5. If $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ $(a, b, c \in R)$ have a common non – real roots then

(A) $-2|a| < b < 2|a|$

(B) $-2|c| < |b| < 2|c|$

(C) $a = \pm c$

(D) $a = c$

6. Consider the equation $x^2 + x - a = 0$, $a \in N$. If equation has real roots then

(A) a = 2 (B) a = 6 (C) a = 12 (D) a = 20

INTEGER TYPE

1. The number of irrational solutions of the equation $\sqrt{x^2+\sqrt{x^2+11}}+\sqrt{x^2-\sqrt{x^2+11}}=4$ is ____

2. Number of real values of x satisfying the equation $\sqrt{x^2-6x+9}+\sqrt{x^2-6x+6}=1$ is ______

3. Find the number of integral values of a for which the system of equations
$\left.\begin{array}{r} x+ay=3 \\ ax+4y=6 \end{array}\right\}$ satisfy $x > 1;\ y > 0$.

4. Minimum value of f(x) = |x - 1| + |2x - 1| + |3x - 1| + |4x - 1| is p/q where p/q is in lowest form and p, q $\in I^+$ then p + q is ________

5. When the polynomial $5x^3 + Mx + N$ is divided by $x^2 + x + 1$, the remainder is 0.
Then the value of |M + N| is _______

6. If a – 2b = 1 then value of $a^3 - 6ab - 8b^3$ is equal to _____

7. The value of $\sqrt{1+5\sqrt{1+......+2013\sqrt{1+2014\sqrt{1+2015\sqrt{1+2016\times 2018}}}}}$ is _______

8. Let r, s, t are roots of equation $8x^3+1001x+2008=0$. Then value of $(r+s)^3+(s+t)^3+(t+r)^3$ is 7k3 (where k is at ten's place). Then value of k is ________

9. If both roots of equation $4x^2-20px+25p^2+15p-66=0$ are greater than 2, then sum of all possible integral values of p is ____

10. Let k be an integer and p is a prime number such that the quadratic equation $x^2+kx+p=0$ has two distinct positive integer solutions. Then the value of $-(p+k)$ is.

11. If the first three consecutive terms of a GP are the real roots of the equation $2x^3-19x^2+57x-54=0$ and k is the sum of infinite number of the terms of this G.P. Then $2k/9$ equals

12. Let $(x + 3)^2 (x + 4)^3 (x + 5)^4 = (x + 1)^9 + a_1 (x + 1)^8 + a_2 (x + 1)^7 + ...+ a_9$ then $a_2 - 365$ is equal to __

SUBJECTIVE PROBLEMS

1. Obtain a polynomial of lowest degree with integral coefficients, whose one of the zero is $\sqrt{5}+\sqrt{2}$.

2. Let P(x) be a polynomial such that $x\cdot P(x-1)=(x-4)\cdot P(x)\ \forall x\in R$. Find all such polynomials

3. Let P(x) be a monic cubic equation such that P(1) = 1, P(2) = 2, P(3) = 3 then find P(4).

4. Show that $f(x) = x^{1000} - x^{500} + x^{100} + x + 1$ has no rational roots.

MATRIX MATCH

1. Match the following

	COULUMN – I		COULUMN - II
A	If a, b, c and d are four non-zero real number such that $(d+a-b)^2+(d+b-c)^2=0$ and the roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are real and equal then	P	$a+b+c=0$
B	If the roots of the equation $(a^2+b^2)x^2-2b(a+c)\ \ +(b^2+c^2)=0$ are real and equal, then	Q	a,b,c are in A.P.

C	If the equation $ax^2+bx+c=0$ and $x^3-3x^2+3x-1=0$ have a common real root, then	R	a,b,c, are in G.P.
D	Let a,b,c be positive real numbers such that the expression $bx^2+\left(\sqrt{(a+c)^2+4b^2}\right)x+(a+c)$ is non-negative $\forall x\in R$, then	S	a,b,c are in H.P.

COMPREHENSION for Q 1 – 3

The first four terms of a sequence are given by $T_1 = 0$, $T_2 = 1$, $T_3 = 1$, $T_4 = 2$. The general term is given by $T_n = A\,\alpha^{n-1} + B\,\beta^{n-1}$ where A, B, α, β are independent of n and A is positive.

1. The value of $(\alpha^2 + \beta^2 + \alpha\beta)$ is equal to

   (A) 1 (B) 2 (C) 5 (D) 4

2. The value of $5(A^2 + B^2)$ is equal to

   (A) 2 (B) 4 (C) 6 (D) 8

3. The quadratic equation whose roots are α and β is given by

   (A) $x^2 - 2x - 1 = 0$ (B) $x^2 - 2x - 2 = 0$ (C) $x^2 - x - 1 = 0$ (D) None

THANKS

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ANSWER KEY & SOLUTION

SINGLE OPTION CORRECT

1. D	2. B	3. D	4. B
5. B	6. D	7. D	8. A
9. B	10.	11.	12.
13. A	14. B	15. C	16. B

17. A, Hint: $P(x) = (x - 1)(x - 2)(x - 3) + x^2 + 2x$

18. A	13. C	14. B	15. C

16. C Hint: $f(x) = 2 + \dfrac{3}{(x^2-4)^2 + (x-2)^2 + 3}$

17. A	18. C

19. C, Hint: AM of roots = HM $\rightarrow \alpha = \beta = \gamma = \delta = 2$. $P = -8$ and $q = 24$

20. C	21. A	22. C	23. A
24. C	25. B	26. D, Hint: $(x - 1)(x - 2)(x^3 - 3x + 1) = 0$	
27. C	28. A	29. B	30. A
31. D	32. B	33. B	34. C
35. C	36. D	37. C	38. C
39. D	40. C	41. A	42. A
43. A			

MULTI OPTIONS CORRECT

1.	2.	3. A, B, C	4. A, C, D
5. A, B, D	6. A, B, C, D		

INTEGER TYPE

1.	2.	3.	4. 7
5. 5	6. 1	7. 6	8. 5
9. 7	10. 1	11. 3	

12. 371

Hint: $x + 1 = y \rightarrow (y + 2)^2 (y + 3)^3 (y + 4)^4 = y^9 + a_1 y^8 + a_2 y^7 + \ldots\ldots + a_9$

a_2 = sum of roots taking two at a time.

SUBJECTIVE

1. $P(x) = a(x^4 - 14x^2 + 9)$, where $a \in I$, $a \neq 0$.
2. $P(x) = c\,x(x - 1)(x - 2)(x - 3)$, $c \neq 0$
3. 10

MATRIX MATCH

1. A→ R, B→ R, C→ P, D → Q

COMPREHENSION for Q 1 - 3

1. B	2. A	3. C