MATHSARC EDUCATION

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MATHEMATICS **LOGARITHM**

SINGLE OPTIONS CORRECT

1. If x, y, z ∈ R+ are such that z > y > x > 1, $\log_y x + \log_x y = 5/2$ and $\log_z y + \log_y z = 10/3$, then $\log_x z$ is equal to –

(A) 2 (B) 3 (C) 6 (D) 12

2. If x, y, z are three positive numbers in G.P., then $\frac{1+\ln x}{2}, \frac{1+\ln y}{4}, \frac{1+\ln z}{8}$ are in

(A) A.P. (B) G.P. (C) H.P. (D) A.G.P.

3. The solution set of the inequality $\sqrt{\log_2 x - 1} + \frac{1}{2}\log_{1/2}\left(x^3\right) + 2 > 0$ is.

(A) [2, 3) (B) (2, 3] (C) [2, 4) (D) (2, 4]

4. If a, b, c are real numbers greater than 1 and
$x = \frac{1}{1+\log_{a^3b^2}\left(c^2/a\right)} + \frac{1}{1+\log_{b^3c^2}\left(a^2/b\right)} + \frac{1}{1+\log_{c^3a^2}\left(b^2/c\right)}$, then 2x is equal to

5. The solution set of the inequation $\log_{1/3}(x^2 + x + 1) + 1 > 0$ is:

(A) $(-\infty, -2) \cup (1, \infty)$ (B) $[-1, 2]$ (C) $(-2, 1)$ (D) R

6. Number of integers which satisfy the inequation $\log_2 \sqrt{\frac{|x|}{2}} < \log_{\left(\frac{|x|}{4}\right)} 2$ is

(A) 4 (B) 6 (C) 8 (D) 10

7. If α, β are the solutions of equation $2^{20+\log_2^2 x} = x^{12}$ such that α < β, then $\frac{\beta}{\alpha}$ is –

(A) 4 (B) 64 (C) 256 (D) 512

8. The sum of all the natural numbers for which $\log_{(4-x)}(x^2 - 14x + 45)$ is defined is –

(A) 1 (B) 2 (C) 3 (D) 4

9. The number of solution(s) of the equation $\log_7(2^x - 1) + \log_7(2^x - 7) = 1$, is

(A) 0 (B) 1 (C) 2 (D) 3

10. If $55^{f(x)} + 5^x - 2012 = 0$ and f(x) is defined. Then possible integral value(s) of x can't be

(A) –1 (B) 2 (C) 3 (D) 5

11. Let **A** denotes the value of $\log_{10}\left(\frac{ab+\sqrt{(ab)^2-4(a+b)}}{2}\right)+\log_{10}\left(\frac{ab-\sqrt{(ab)^2-4(a+b)}}{2}\right)$ where a = 43, b = 57 and **B** denotes the value of the expression $\left(2^{\log_6 18}\right)\times\left(3^{\log_6 3}\right)$. Find the value of (A×B).

(A) 12 (B) 18 (C) 43 (D) None of these

12. The increasing geometric sequence $x_0, x_1, x_2, \ldots\ldots$ consists entirely of integral power of 3. Given that $\sum_{n=0}^{7}\log_3(x_n)=308$ and $56\le\log_3\left(\sum_{n=0}^{7}x_n\right)\le 57$, then the value of $\log_3(x_{14})$ is equal to ____

(A) 70 (B) 91 (C) 112 (D) 133

13. Let $f(x)=\left(x^2+3x+2\right)^{\cos(\pi x)}$. Find the sum of all positive integers n for which $\left|\sum_{k=1}^{n}\log_{10} f(k)\right|=1$

(A) 18 (B) 19 (C) 20 (D) 21

14. The system of equations

$$\log_{10}(2000xy)-(\log_{10}x)(\log_{10}y)=4$$
$$\log_{10}(2yz)-(\log_{10}y)(\log_{10}z)=1$$
$$\log_{10}(zx)-(\log_{10}z)(\log_{10}x)=0$$

Has two solutions (x_1, y_1, z_1) and (x_2, y_2, z_2) then the value of $y_1 + y_2$ is __

(A) 20 (B) 25 (C) 100 (D) 101

MULTIPLE OPTIONS CORRECT

1. If $6^{x+1}=2^{3x+1}$, then x is equal to –

(A) $\frac{\log_2 3}{2-\log_2 3}$ (B) $\log_{4/3}3$ (C) $\log_{3/4}3$ (D) $\frac{1}{1-2\log_3 2}$

2. Let S = $\log_a bc + \log_b ca + \log_c ab$ where a, b, c are real numbers greater than 1, then 'S' can be equal to –

(A) 4 (B) 6 (C) 3 (D) 8

SUBJECTIVE PROBLEMS

1. If $\log_4(x + 2y) + \log_4(x - 2y) = 1$, then the minimum value of $|x| - |y|$ is ______
2. Let a, b, c, d be positive integers and $\log_a b = 3/2$, $\log_c d = 5/4$. If a – c = 9, then b – d = _____
3. Evaluate the following

(i) $\log_{64}512$ (ii) $\log_{10}(0.01)$ (iii) $\log_{19}6859$ (iv) $\log_2\left(16^{1/3}\times\sqrt{8}\right)$

(v) $\log_2\left(\frac{\sqrt[3]{4}}{4^2\sqrt{8}}\right)$ (vi) $2^{-\frac{\log_2 64}{3}}$

4. Simplify the following:

(i) $4^{5\log_{4\sqrt{2}}\left(3-\sqrt{6}\right)-6\log_8\left(\sqrt{3}-\sqrt{2}\right)}$

(ii) $\dfrac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\times\left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25} 7}}-(125)^{\log_{25} 6}\right)$

(iii) $5^{\log_{1/5}\left(\frac{1}{2}\right)}+\log_{\sqrt{2}}\dfrac{4}{\sqrt{7}+\sqrt{3}}+\log_{1/2}\dfrac{1}{10+2\sqrt{21}}$

(iv) $49^{(1-\log_7 2)}+5^{-\log_5 4}$

(v) $\log_{1/3}\sqrt[4]{729\cdot\sqrt[3]{9^{-1}\times 27^{-4/3}}}$

(vi) $a^{\frac{\log_b(\log_b N)}{\log_b a}}$

(vii) $\dfrac{2}{\log_4(2000)^6}+\dfrac{3}{\log_5(2000)^6}$

5. Prove that

(i) log(1 + 2 + 3) = log 1 + log 2 + log 3

(ii) log 360 = 3 log 2 + 2 log 3 + log 5

(iii) $\log\left(\dfrac{50}{147}\right)=\log 2+2\log 5-\log 3-2\log 7$

(iv) log40 = 3 log 2 + log 5

(v) log 2178 = 5 log3 + log 9

(vi) $\log\dfrac{a^2}{bc}+\log\dfrac{b^2}{ac}+\log\dfrac{c^2}{ab}=0$

(vii) $7\log\dfrac{16}{25}+5\log\dfrac{25}{24}+3\log\dfrac{81}{80}=\log 2$

(viii) $3\log\dfrac{49}{16}+\dfrac{5}{2}\log\dfrac{256}{81}-2\log\dfrac{343}{243}=8\log 2$

(ix) $\dfrac{\log_2 24}{\log_{96} 2}-\dfrac{\log_2 192}{\log_{12} 2}=3$

6. (i) If $x=\log_3 4$ & $y=\log_5 3$. Find the value of $\log_3 10$ & $\log_3(6/5)$ in terms of x and y.

(ii) If $k^{\log_2 5}=16$, find the value of $k^{(\log_2 5)^2}$

7. (i) If $\log\dfrac{a+b}{3}=\dfrac{1}{2}(\log a+\log b)$. Show that $a^2+b^2=7ab$.

(ii) If $x^2+y^2=23xy$ show that $\log\left(\dfrac{x+y}{5}\right)=\dfrac{1}{2}(\log x+\log y)$.

8. Given $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$, Evaluate $\log(36)^{1/4}$.

LOGARITHMIC EQUATIONS

1. $\log_{x-1}3=2$

2. $\log_3(3^x-8)=2-x$

3. $\log_3(x+1)+\log_3(x+3)=1$

4. $\log_7(2^x-1)+\log_7(2^x-7)=1$

5. $\log_3(1+\log_3(2^x-7))=1$

6. $9^{\log_3(1-2x)}=5x^2-5$

7. $x^{2\log x}=10x^2$

8. $3\sqrt{\log_2 x}-\log_2 8x+1=0$

9. $\log^2 x - 3\log x = \log(x^2) - 4$

10. $2\log_3 \frac{x-3}{x-7} + 1 = \log_3 \frac{x-3}{x-1}$

11. $\log_{\frac{1}{3}}(5x-1) > 0$

12. $\log_3(3x-1) < 1$

13. $\log_{0.5}\left(x^2 - 5x + 6\right) > -1$

14. $\log_8\left(x^2 - 4x + 3\right) \le 1$

15. $\log_7 \frac{2x-6}{2x-1} > 0$

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THANKS!

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ANSWER KEY

SINGLE OPTION CORRECT

1. C 2. D 3. C

10 D

$55^{f(x)} = 2012 - 5^x \Rightarrow f(x) = \log_{55}(2012 - 5^x)$

for f(x) to be defined $2012 - 5^x > 0$

$\Rightarrow 5^x < 2012 \Rightarrow x < 5$ [since $5^4 < 2012 < 5^5$]

$\therefore$ Integral value of x = –1, 2, 3

11. A 12. B 13. D 14. B

MULTIPLE OPTION CORRECT

1. A, B 2. B, D

SUBJECTIVE PROBLEMS

1. $\sqrt{3}$ 2. 93

3. (i)1.5 (ii) - 2 (iii) 3 (iv) 17/6

(v) - 29/6 (vi) 1/4

4. (i) 9 (ii) 1 (iii) 6 (iv) 25/2

(v)- 1 (vi) $\log_b N$ (vii) 1/6

6. (i) $\dfrac{xy+2}{2y}, \dfrac{xy+2y-2}{2y}$ (ii) 625

8. 0.3891

LOGARITHMIC EQUATIONS

1. $\{1+\sqrt{3}\}$ 2. {2} 3. {0} 4. {3}

5. {4} 6. $\{-2-\sqrt{10}\}$ 7. $\left\{\sqrt{10^{1-\sqrt{3}}}, \sqrt{10^{1+\sqrt{3}}}\right\}$

8. {2, 16} 9. $\{10, 10^4\}$ 10. {–5} 11. $\left(\dfrac{1}{5}, \dfrac{2}{5}\right)$

12. (1/3,2) 13. $(1,2)\cup(3,4)$ 14. $[-1,1)\cup(3, 5]$ 15. $\left(-\infty, \dfrac{1}{2}\right)$